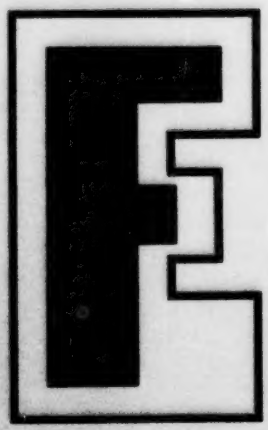

Economics & Forecasts Division
Corporate Planning Branch

PREDICTION RANGES
ANNUAL REVIEW

September 1988

This review replaces the September 1987 Prediction Ranges.

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Note to Users: The Economics and Forecasts Division provides two types of uncertainty measures. The alternative scenarios, described in another report, are to be used in instances where you have a planning model which responds to changes in a large set of economic variables over the course of several years. The uncertainty measure described in this report consists of specific values related to the statistical probability distribution of a particular variable. They should be used when your plan depends on the outcome of a key economic variable at a specific point in time, and you want to be able to specify the probability range of the outcomes of your planning goal.

1.0 INTRODUCTION

The Economics and Forecasts Division produces specific numerical values for its forecast variables, but these are not intended to be taken as precisely accurate. Rather, they are best interpreted to represent the weighted average of a range of probable outcomes. The exact values presented are unlikely to occur, and will not except by chance. Where results are not very sensitive to economic assumptions, these point forecasts may be sufficient for corporate planning purposes. However, most decisions or plans are highly sensitive to certain economic indicators. For example, a project judged to be cost effective when the interest rate is forecast to be 10% could turn out not to be viable if a 12% rate is used instead. Moreover, from the forecaster's point of view, the probability of a 12% rate might well be just a touch lower than that of the official forecast of 10%. The idea of the prediction range therefore is to equip the planner (decision maker) with one more tool so that he can better assess the outcome of a course of action.

Unlike the alternative scenarios, which make specific alternative assumptions about demography, government policy and so on, the prediction ranges take as given the assumptions underlying the most likely scenario and carefully assess the risks and uncertainties associated with them. The result is upper and lower bounds around the point forecast of the most likely. A range with 50% probability implies an equal chance that the realized value will fall within the bounds or outside them. Probability or confidence rises as the bounds spread farther apart. Thus an 80% range is expected to include the actual future value eight times out of ten. High confidence is not necessarily a

desirable property by itself, however, because a prediction range is of little use to the planner if it gets too wide.

Confidence in the present context is not to be confused with the confidence level found in statistics. The latter refers to an interval statistically calculated in a mechanistic fashion. Indeed, the forecaster may want to calculate such an interval insofar as data is available before drawing on analytical inferences and his own judgment. In this connection, the Economics and Forecasts Division has developed a simple model to compute the confidence intervals (in the statistical sense) for Canada and another for the United States. They are used as an input to prepare prediction ranges for both the Canadian and the United States sections. But in the end, probability as used in this document is largely a measure of the subjective confidence that the forecaster attaches to a prediction range. (The meaning of this probability is further explored in the Appendix.)

Each economic variable in the document is supplied with two ranges. The 50% range is furnished by the analyst responsible for the variable. From this is calculated the standard error, which is required by some users. In addition, the 50% range is used to generate the 80% range. The result is two internally consistent prediction ranges.

2.0 APPLICATIONS

Point forecasts, which are widely used across the corporation, are prone to error. The prediction ranges alert the corporate planner to this and provide a range of possible values. He can evaluate the sensitivity of his plan to variations in the economic variables and where feasible and practical should take appropriate action to minimize its impact.

Any scenario implicitly has an uncertainty range around it. Only ranges of the most likely are produced below since this is the corporation's central planning scenario. Every time major decisions are to be made and cost-benefit studies are to be carried out with the most likely, the prediction ranges will be

indispensable. In fact, one of their major uses has been in financial evaluations where corporate policy requires the performance of uncertainty analysis for large projects. They are used as inputs to the "Monte Carlo" simulation, which is a statistical technique of repeated experiments with the data to simulate the likely distribution of outcomes. Furthermore, prediction ranges with 80% probability may be construed as the upper- and lower-limits of variables in an uncertain environment. Using the extreme values, the planner can assess the most favourable case (or conversely the worst case) and get an idea of the potential impacts.

As a general rule, prediction ranges are appropriate for sensitivity studies relating to a single economic variable, or for a random sampling (ie, Monte Carlo) simulation of a group of variables. Where a whole set of variables interacting with each other is required, alternative scenarios are more applicable.

2.1 Examples

The prediction ranges have wide applications:

Budgeting In 1985, Ontario Hydro and its union agreed to a new contract which would give union members a 3.5% wage increase in the second year of the two-year contract. It also provided an additional 1% increase for each percentage point the consumer price index rose above 3.7% in 1986. The Corporation therefore needed to know how much this cost-of-living allowance might eventually cost so it could budget accordingly. Using the point forecast at the time, which had inflation for 1986 at around 4.5%, an additional 1% increase in the union wage budget would appear appropriate. A more cautious approach, however, would suggest an additional 2% because according to the prediction ranges there was a good chance that the rate of inflation could be as high as 5.5% in 1986. (As it turned out, the inflation rate of 4.1% was too low to trigger the cost-of-living clause.)

Revenue Impact Ontario Hydro's revenue requirements, and therefore the real price of electricity, are highly sensitive to economic factors outside Ontario

Hydro's control. This is discussed in Chapter 7 of the Financial Forecast, 1986-1989, issued by Comptroller's Division. That chapter provides an example of how prediction ranges and similar information can be used to assess the relative risks of desirable versus undesirable uncertainties.

Bidding From time to time Ontario Hydro invites foreign suppliers to bid on contracts. In a situation where there are two close bids with one each coming from domestic and foreign sources, foreign exchange exposure has to be taken into account. Suppose within the 50% range the value of the foreign currency relative to the Canadian dollar can be 10% higher or 5% lower than the point forecast. Based on the sensitivity analysis it is revealed that the foreign bid could be actually $1 million lower than the domestic bid, or it could be $2 million higher. It then may be concluded that the domestic source is preferable given the upside risk of the foreign bid.

Borrowing Interest rates are expected to trend downward over the longer run. Suppose Ontario Hydro is considering whether to borrow long or short over a ten-year period. If a ten-year bond is issued, the corporation is assured of a steady rate but will not be able to take advantage of the expected dip in interest rates. On the other hand, the corporation can issue a five-year bond and re-finance it at a lower rate five years hence. But there is a risk attached to short-term financing in that the interest rate may end up higher in five years' time than it is currently. Once again the prediction ranges are fundamental to this kind of analysis.

3.0 RISKS AND UNCERTAINTIES

Relatively little is known about the future. Assessment of future uncertainties therefore is highly subjective and can differ greatly among individual forecasters. This section outlines the major issues as perceived by the Economics and Forecasts Division which are reflected in the prediction ranges that follow.

The major short-term risks relate to inflationary and political risks. The coming Canadian and US federal elections increase the risk of policy shifts in the next five years. Longer term growth uncertainties stem from potential changes in demographics and productivity growth.

3.1 Growth

The most likely forecast has similar demographic trends in Ontario, Canada and the United States. Population and labour force growth rates are expected to slow. The proportion of the population under 20 will fall and that greater than 65 will rise.

Labour force growth could become negative around 2015. The aging of the population may decrease labour mobility. These two factors could produce labour shortages. The most likely growth forecast is driven in the long run by these demographic factors, changes in productivity and changes in the structure of the economy. While we are fairly certain about the number of people who will be 65 around 2010-2015 (barring any large changes in old age immigration) there are uncertainties attached to the forecast.

The North American economies could receive a boost to their growth from four main areas. First, as the baby boom generation gains experience we could see productivity gains. Baby boomers invested heavily in training and education and in the next 10 to 15 years the economy could reap the benefits of this. Secondly, a labour shortage could lead to a more efficient use of the existing labour supply. Thirdly, if the North American economies can attract more immigrants they can augment their dwindling labour force growth rates. Productivity could rise as immigrants tend to be better educated and trained and have a higher labour force participation than others. Another growth benefit could come from the increased demand for goods and services by immigrants. Finally, fertility rates might not remain at their current low levels and population could once again start to grow.

There are down-side risks as well. The lower labour force growth and fewer young people could lead to a less flexible, less mobile work force. The

slowing population growth could have a dampening effect on aggregate demand in the economy if the economy is presently over-built in anticipation of higher growth rates. Also, it may take industries time to adjust to the changing demands of an aging population.

Also contributing to the risk of lower growth is North America's ability to continue to attract immigrants. Canada has had an on-off immigration policy in the past. While immigration policy seems to have loosened up in the last year it is subject to political changes.

There is a risk for the Ontario economy in that Ontario's share of immigration has been historically related to Ontario's economic performance relative to Canada's. We are forecasting only slightly higher growth for Ontario than Canada implying that Ontario will become a relatively less attractive home for immigrants than it has been recently.

Finally, the aging population could cause a shift in resources to the government sector because of the increase in the number of old people. The higher cost of supporting people over 65 relative to those under 20 means that the falling proportion of young people might not offset the effect of aging on the demand for government services. The upshot is that governments could be forced to run deficits to finance the services provided to the old. These deficits could provide a drag on the North American economy. While this is a risk it is not included in the most-likely because we believe government revenues will rise as a result of two factors. First, as the baby boom generation reaches its peak earnings, income tax collections will rise. Secondly, as this group moves beyond the house buying age, they will spend more money on luxury-type items and so sales tax revenue should rise. Should these revenues not materialize, governments could be tempted to inflate when faced with unfunded pensions and higher health care costs.

Two other risks for the Canadian and Ontario economies relate to the United States. First, the free trade agreement is subject at the time of writing to some political risk in Canada. The most-likely case incorporates the expected impacts of the free trade agreement. A cancellation will lower growth in

Ontario and Canada due to lower productivity. Canada might also be subject to increased protectionism should the free trade agreement not be passed by government. Any protectionist moves by the United States would seriously affect Canadian and Ontario economic growth.

The US economy is facing a problem with its twin (government and external trade) deficits. The impact of these deficits on future monetary and fiscal policy is unknown and adds to the risk associated with the medium term growth prospects. In general terms, a US downturn could have a larger impact on the Canadian and Ontario economies than that assumed in the most-likely case. Canada and Ontario are heavily dependent on exports to the United States for their growth. The size of the downturn in Canada depends, in part, upon fiscal and monetary policies.

3.2 Inflation

The current recovery has been extremely long. As a result capacity utilization is generally high across the economy. Central Banks have expressed concern that their economies are overheating and interest rates have increased to try to contain inflationary pressures.

The risk on the inflationary front is that the response of monetary authorities is uncertain. On one hand they may try to use the generally healthy economies as a cushion to try to squeeze inflation to much lower levels. The most-likely forecast suggests that the depth and extent of the 1981-2 recession could make policy makers tread a more cautious path. Should governments want to prolong the current recovery a few years they may be tempted to allow inflation to creep up.

The US deficit remains a problem on the inflation front. There is an incentive for some future US government to inflate the problem away. The United States is also in poorer financial shape than Canada: the economy is more highly leveraged and the banking system is on less sure footing. If the United States does follow a more inflationary path than Canada there would be an exchange rate adjustment. This is discussed in the next section.

3.3 Interest Rates and Exchange Rates

Over the next few years, there is considerable uncertainty about the interaction of monetary and fiscal policy and how this will affect interest rates and economic activity. Excessively large tax cuts in the United States have led to massive government deficits and raised interest rates worldwide. If the US government lowered its spending or raised taxes, interest rates could be lower. Conversely, if the central bank maintained its policy of restraining money supply growth, real interest rates could remain high.

The value of the US dollar will be affected by the change in the government and current account deficits. Any improvement here would decrease the need for foreign (non US) savings and lead to greater stability for the US dollar.

The values of other currencies relative to the US dollar will also be considerably affected by interest rate developments, since they tend to be weakened by high US interest rates. However, deteriorating confidence in US economic policies may lead to a combination of stronger foreign currencies despite higher interest rates.

For the longer term, the greatest uncertainty affecting interest rates is the level and oscillation of the rate of inflation. A highly variable rate of inflation increases the riskiness of investing in bonds and tends to drive up interest rates. Exchange rates in the long term are also substantially determined by the relative price levels in the different countries. The most-likely forecast assumes that Canada will, on average, have slightly higher inflation than the United States, causing the Canadian dollar to have a mild downward trend. An even higher inflation differential could cause the dollar to fall faster. Conversely, if Canada had a persistently lower inflation rate than the United States, our dollar would rise in value. Exchange rates are highly uncertain because a small difference in annual inflation rates can cumulate over a number of years into a large difference in the relative levels of the values of the currencies.

4.0 PREDICTION RANGES AND STANDARD DEVIATIONS

4.1 Ontario

Real GDP(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	3.3	3.7	3.9	4.3	4.7	0.444
1989	1.2	2.0	2.7	3.2	4.0	0.889
1990	-0.3	1.0	2.3	3.0	4.3	1.481
1991	0.7	2.0	3.5	4.1	5.4	1.556
1992	1.0	2.3	3.8	4.4	5.7	1.556
1991-1995	1.4	2.2	3.2	3.5	4.3	0.963
1996-2000	1.2	2.0	2.8	3.3	4.1	0.963
2001-2005	1.1	1.9	2.4	3.2	4.0	0.963
2006-2010	0.9	1.7	2.1	3.0	3.8	0.963
2011-2025	0.4	1.4	1.7	3.0	4.0	1.185

Ontario CPI(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	3.9	4.3	4.5	5.0	5.4	0.519
1989	1.9	3.5	4.8	6.0	7.6	1.852
1990	2.6	3.6	4.6	5.2	6.2	1.185
1991	1.6	3.1	4.5	5.5	7.0	1.778
1992	1.6	3.3	4.5	6.0	7.7	2.000
1991-1995	1.2	3.1	4.5	6.0	7.9	2.148
1996-2000	0.4	3.0	5.0	7.0	9.6	2.963
2001-2005	0.4	3.0	5.5	7.0	9.6	2.963
2006-2010	0.4	3.0	5.0	7.0	9.6	2.963
2011-2025	0.4	3.0	5.0	7.0	9.6	2.963

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1. Annual Average percent change

4.2 Canada

Real GDP(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	3.2	3.6	3.8	4.2	4.6	0.444
1989	1.0	1.8	2.5	3.0	3.8	0.889
1990	-0.6	0.7	2.1	2.8	4.1	1.556
1991	-0.2	1.5	3.2	4.2	5.9	2.000
1992	0.7	2.0	3.4	4.1	5.4	1.556
1991-1995	1.4	2.4	3.2	3.9	4.9	1.111
1996-2000	0.9	2.0	2.6	3.7	4.8	1.259
2001-2005	1.4	2.0	2.6	3.0	3.6	0.741
2006-2010	0.8	1.6	2.1	2.8	3.6	0.889
2011-2025	0.3	1.3	1.7	2.8	3.8	1.111

Average Hourly Earnings(1)
Manufacturing

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	2.4	4.0	4.5	6.5	8.1	1.852
1989	2.2	3.9	5.0	6.5	8.2	1.926
1990	1.9	3.6	5.0	6.3	8.0	2.000
1991	1.4	3.3	5.0	6.3	8.2	2.222
1992	1.6	3.5	5.0	6.5	8.4	2.222
1991-1995	2.1	4.0	5.3	7.0	8.9	2.222
1996-2000	1.4	4.2	5.9	8.5	11.3	3.185
2001-2005	1.3	4.2	7.0	8.7	11.6	3.333
2006-2010	1.3	4.2	7.0	8.7	11.6	3.333
2011-2025	1.3	4.2	6.5	8.7	11.6	3.333

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1. Annual average percent change

4.2 Canada (continued)

Employment(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	1.2	2.5	2.9	4.5	5.8	1.481
1989	-0.1	1.5	1.8	4.0	5.6	1.852
1990	-2.0	-0.2	1.7	2.6	4.4	2.074
1991	-0.7	1.2	2.0	4.1	6.0	2.148
1992	-0.8	0.4	1.3	2.3	3.5	1.407
1991-1995	-0.3	0.7	1.4	2.3	3.3	1.185
1996-2000	-0.5	0.5	0.8	2.0	3.0	1.111
2001-2005	-0.7	0.3	0.6	1.8	2.8	1.111
2006-2010	-0.9	0.1	0.4	1.6	2.6	1.111
2011-2025	-1.5	-0.5	0.0	1.0	2.0	1.111

Labour Productivity(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	0.2	0.6	0.8	1.2	1.6	0.444
1989	-0.5	0.3	0.7	1.5	2.3	0.889
1990	-1.5	0.0	0.4	2.3	3.8	1.704
1991	-1.1	0.3	1.3	2.5	3.9	1.630
1992	-0.8	0.5	2.1	2.5	3.8	1.481
1991-1995	0.0	1.0	1.8	2.5	3.5	1.111
1996-2000	0.0	1.0	1.8	2.5	3.5	1.111
2001-2005	0.0	1.0	2.0	2.5	3.5	1.111
2006-2010	0.0	1.0	1.7	2.5	3.5	1.111
2011-2025	0.0	1.0	1.7	2.5	3.5	1.111

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1. Annual average percent change

4.2 Canada (continued)

Prime Rate(2)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	10.0	10.3	10.5	10.9	11.4	0.444
1989	6.4	9.0	11.0	13.0	15.6	2.963
1990	6.1	8.0	9.5	12.0	15.2	2.963
1991	6.7	8.0	9.0	12.0	15.8	2.963
1992	6.1	8.0	9.5	12.0	15.2	2.963
1991-1995	5.6	7.5	9.0	12.0	15.8	3.333
1996-2000	5.6	7.5	9.0	11.5	14.7	2.963
2001-2005	5.6	7.5	9.0	11.5	14.7	2.963
2006-2010	5.1	7.0	8.5	11.5	15.3	3.333
2011-2025	4.6	6.5	8.0	11.5	16.0	3.704

Intermediate Government(2) Bond Rate

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	9.0	9.4	9.7	10.0	10.4	0.444
1989	7.8	9.0	9.9	11.0	12.4	1.481
1990	6.8	7.8	8.5	11.0	14.2	2.407
1991	6.2	7.5	8.5	11.0	14.2	2.593
1992	5.6	7.5	9.0	11.0	13.6	2.593
1991-1995	5.6	7.5	9.0	11.5	14.7	2.963
1996-2000	5.6	7.5	9.0	11.5	14.7	2.963
2001-2005	5.6	7.5	9.0	11.5	14.7	2.963
2006-2010	5.1	7.0	8.5	11.5	15.3	3.333
2011-2025	4.6	6.5	8.0	11.5	16.0	3.704

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2. Percent

4.2 Canada (continued)

YEAR	Long-Term Government(2) Bond Rate 80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	9.6	10.0	10.3	10.6	11.0	0.444
1989	7.2	9.0	10.4	11.2	12.2	1.630
1990	7.9	8.5	9.0	11.2	14.0	2.000
1991	6.7	8.0	9.0	12.0	15.8	2.963
1992	6.1	8.0	9.5	12.0	15.2	2.963
1991-1995	6.5	8.0	9.2	12.0	15.6	2.963
1996-2000	6.1	8.0	9.5	12.0	15.2	2.963
2001-2005	6.1	8.0	9.5	12.0	15.2	2.963
2006-2010	5.6	7.5	9.0	12.0	15.8	3.333
2011-2025	5.1	7.0	8.5	12.0	16.5	3.704

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2. Percent

4.3 United States

Real GNP(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	2.9	3.3	3.5	3.9	4.3	0.444
1989	1.0	1.8	2.5	3.0	3.8	0.889
1990	-0.8	0.5	1.8	2.5	3.8	1.481
1991	-1.2	1.0	3.0	4.5	6.7	2.593
1992	0.2	1.5	2.7	3.5	4.8	1.481
1991-1995	0.7	1.7	2.5	3.2	4.2	1.111
1996-2000	1.4	2.0	2.5	3.0	3.6	0.741
2001-2005	1.4	2.0	2.5	3.0	3.6	0.741
2006-2010	1.2	1.8	2.3	2.8	3.4	0.741
2011-2025	0.4	1.5	2.0	3.2	4.3	1.259

CPI(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	3.6	3.9	4.1	4.3	4.6	0.296
1989	2.0	3.6	4.9	6.1	7.7	1.852
1990	2.1	3.4	4.4	5.5	6.8	1.556
1991	1.1	2.8	3.9	5.5	7.2	2.000
1992	1.6	3.3	4.5	6.0	7.7	2.000
1991-1995	1.1	3.0	4.5	6.0	7.9	2.222
1996-2000	0.4	3.0	5.0	7.0	9.6	2.963
2001-2005	0.4	3.0	5.0	7.0	9.6	2.963
2006-2010	0.4	3.0	5.0	7.0	9.6	2.963
2011-2025	0.4	3.0	5.0	7.0	9.6	2.963

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1. Annual average percent change

4.3 United States (continued)

Prime Rate(2)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	8.6	9.0	9.3	9.9	10.7	0.667
1989	6.4	8.5	10.1	12.5	15.6	2.963
1990	5.2	7.0	8.4	10.5	13.2	2.593
1991	4.7	6.5	7.9	10.5	13.8	2.963
1992	4.2	6.5	8.3	10.5	13.3	2.963
1991-1995	5.0	6.9	8.4	11.0	14.3	3.037
1996-2000	3.9	6.5	8.5	11.0	14.2	3.333
2001-2005	3.4	6.0	8.0	11.0	14.8	3.704
2006-2010	3.7	6.0	7.8	11.0	15.1	3.704
2011-2025	2.9	5.5	7.5	11.0	15.5	4.074

Long-Term Government Bonds(2)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	8.5	8.9	9.2	9.6	10.1	0.519
1989	4.9	7.5	9.5	11.5	14.1	2.963
1990	5.7	7.0	8.0	11.0	14.8	2.963
1991	5.6	7.0	8.1	11.5	15.9	3.333
1992	5.1	7.0	8.5	11.0	14.2	2.963
1991-1995	3.9	6.5	8.5	11.0	14.2	3.333
1996-2000	3.9	6.5	8.5	11.0	14.2	3.333
2001-2005	4.3	6.5	8.2	11.0	14.6	3.333
2006-2010	4.1	6.3	8.0	11.0	14.8	3.481
2011-2025	2.9	5.5	7.5	11.0	15.5	4.074

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2. Percent

4.4 Foreign Exchange (Average Values Per Period)

U.S.$/Cdn.$

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	0.785	0.795	0.805	0.810	0.820	0.011
1989	0.715	0.760	0.800	0.830	0.875	0.052
1990	0.722	0.760	0.790	0.820	0.858	0.044
1991	0.722	0.760	0.780	0.820	0.858	0.044
1992	0.722	0.760	0.780	0.820	0.858	0.044
1991-1995	0.669	0.740	0.790	0.850	0.921	0.081
1996-2000	0.617	0.720	0.800	0.880	0.983	0.119
2001-2005	0.565	0.680	0.780	0.860	0.975	0.133
2006-2010	0.532	0.660	0.770	0.860	0.988	0.148
2011-2025	0.433	0.600	0.750	0.860	1.027	0.193

Cdn.$/Deutschemark

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	0.666	0.685	0.700	0.715	0.734	0.022
1989	0.515	0.610	0.671	0.758	0.853	0.110
1990	0.498	0.633	0.723	0.844	0.979	0.156
1991	0.504	0.641	0.754	0.855	0.992	0.159
1992	0.465	0.641	0.777	0.916	1.092	0.204
1991-1995	0.537	0.674	0.763	0.887	1.024	0.158
1996-2000	0.539	0.720	0.816	1.003	1.184	0.210
2001-2005	0.548	0.760	0.869	1.090	1.302	0.244
2006-2010	0.504	0.800	0.926	1.261	1.557	0.341
2011-2025	0.400	0.850	1.092	1.661	2.181	0.601

4.4 Foreign Exchange (Average Values Per Period) (continued)

Cdn.$/Swiss Franc

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	0.793	0.819	0.839	0.859	0.885	0.030
1989	0.634	0.735	0.802	0.893	0.994	0.117
1990	0.634	0.767	0.861	0.974	1.107	0.153
1991	0.617	0.777	0.897	1.026	1.186	0.184
1992	0.590	0.777	0.916	1.068	1.255	0.216
1991-1995	0.675	0.817	0.893	1.038	1.180	0.164
1996-2000	0.630	0.842	0.954	1.173	1.385	0.245
2001-2005	0.639	0.886	1.013	1.271	1.518	0.285
2006-2010	0.590	0.937	1.085	1.478	1.825	0.401
2011-2025	0.480	0.995	1.278	1.943	2.551	0.702

Cdn.$/United Kingdom Pound

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	2.142	2.180	2.210	2.240	2.278	0.044
1989	1.722	1.940	2.130	2.280	2.498	0.252
1990	1.475	1.840	2.090	2.410	2.775	0.422
1991	1.418	1.790	2.050	2.370	2.742	0.430
1992	1.457	1.790	2.050	2.310	2.643	0.385
1991-1995	1.333	1.750	2.020	2.400	2.817	0.481
1996-2000	1.250	1.700	2.030	2.500	3.013	0.593
2001-2005	1.150	1.650	2.100	2.750	3.455	0.815
2006-2010	1.020	1.550	2.130	2.950	3.847	1.037
2011-2025	0.700	1.450	2.190	3.500	4.814	1.519

4.4 Foreign Exchange (Average Values Per Period) (continued)

Cdn.$/100 Japanese Yen

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	0.937	0.950	0.960	0.970	0.983	0.015
1989	0.797	0.893	0.929	1.042	1.138	0.110
1990	0.756	0.873	0.974	1.055	1.172	0.135
1991	0.736	0.884	0.986	1.115	1.263	0.171
1992	0.703	0.884	0.986	1.166	1.347	0.209
1991-1995	0.750	0.900	0.969	1.200	1.392	0.222
1996-2000	0.780	0.950	1.041	1.400	1.688	0.333
2001-2005	0.800	0.950	1.091	1.550	1.935	0.444
2006-2010	0.750	0.975	1.133	1.650	2.083	0.500
2011-2025	0.600	1.000	1.230	3.100	4.446	1.556

4.5 Ontario Hydro

Interest Rates: Longs in the United States(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	9.3	9.7	10.0	10.4	10.9	0.519
1989	5.7	8.3	10.3	12.3	14.9	2.963
1990	6.5	7.8	8.8	11.8	15.6	2.963
1991	6.4	7.8	8.9	12.3	16.7	3.333
1992	5.9	7.8	9.3	11.8	15.0	2.963
1991-1995	4.5	7.1	9.1	11.6	14.8	3.333
1996-2000	4.4	7.0	9.0	11.5	14.7	3.333
2001-2005	4.8	7.0	8.7	11.5	15.1	3.333
2006-2010	4.6	6.8	8.5	11.5	15.3	3.481
2011-2025	3.4	6.0	8.0	11.5	16.0	4.074

Interest Rates: Longs in Canada(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	10.0	10.4	10.7	11.0	11.4	0.444
1989	7.7	9.5	10.9	11.7	12.7	1.630
1990	8.5	9.1	9.6	11.7	14.4	1.926
1991	7.2	8.6	9.7	12.5	16.1	2.889
1992	6.7	8.6	10.1	12.5	15.6	2.889
1991-1995	7.1	8.6	9.8	12.5	16.0	2.889
1996-2000	6.6	8.5	10.0	12.5	15.7	2.963
2001-2005	6.6	8.5	10.0	12.5	15.7	2.963
2006-2010	6.1	8.0	9.5	12.5	16.3	3.333
2011-2025	5.6	7.5	9.0	12.5	17.0	3.704

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1. Percent

4.5 Ontario Hydro (continued)

Interest Rates: Intermediates in the United States(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	8.5	8.9	9.2	9.6	10.1	0.519
1989	5.0	7.6	9.6	11.6	14.2	2.963
1990	5.8	7.1	8.1	11.1	14.9	2.963
1991	5.6	7.1	8.3	11.7	16.1	3.407
1992	5.0	7.1	8.7	11.2	14.4	3.037
1991-1995	3.9	6.5	8.5	11.0	14.2	3.333
1996-2000	3.9	6.5	8.5	11.0	14.2	3.333
2001-2005	4.3	6.5	8.2	11.0	14.6	3.333
2006-2010	4.1	6.3	8.0	11.0	14.8	3.481
2011-2025	2.9	5.5	7.5	11.0	15.5	4.074

Interest Rates: Intermediates in Canada(1)

YEAR	80%	50%	MOST LIKELY	50%	80%	STANDARD ERROR
1988	9.3	9.7	10.0	10.3	10.7	0.444
1989	7.1	8.9	10.3	11.1	12.1	1.630
1990	7.8	8.4	8.9	11.0	13.7	1.926
1991	6.6	8.0	9.1	11.9	15.5	2.889
1992	6.1	8.0	9.5	11.9	15.0	2.889
1991-1995	6.1	8.0	9.5	12.0	15.2	2.963
1996-2000	6.1	8.0	9.5	12.0	15.2	2.963
2001-2005	6.1	8.0	9.5	12.0	15.2	2.963
2006-2010	5.6	7.5	9.0	12.0	15.8	3.333
2011-2025	5.1	7.0	8.5	12.0	16.5	3.704

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1. Percent

APPENDIX: AN EXPLANATORY NOTE ON PROBABILITY

The probability associated with a prediction range of a variable can be thought as **a reasonable degree of belief,** which satisfies certain rules of consistency and is used to describe the variable's behaviour under uncertainty.[1] The degree of reasonable belief depends on the state of one's current knowledge (information). Therefore, in general, a probability representing a degree of reasonable belief is always a conditional probability; that is, it is conditional on the forecaster's present state of information. For instance, the probability assessments provided in this document are based upon (or conditioned on) such presumptions as the absence of a war, normal environmental conditions and the like. As new information is received, the forecaster may revise his belief and the probability assigned to it. This process of revising probabilities in the light of new information is the essence of learning from experience. However, since at any point in time the state of information is given, this conditional probability can be simply called **probability** without ambiguity. In this document, therefore, we use probability rather than "conditional probability" to describe the forecaster's degree of reasonable belief in general. This will also avoid possible confusion with conditional probabilities as defined in a model (see the example below).

The probability assigned to the prediction range of a particular variable should be used separately and independently of those of other variables. Technically, this is called the **marginal probability** of that variable, which is a weighted average of the conditional probabilities of that variable given all the possible outcomes of other variables. The weights are the probabilities of these given possible outcomes.

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[1] cf. Harold Jeffreys, **Theory of Probability** (3rd edition), Oxford: Clarendon, 1961.

The above description can be summarized by (assuming n possible outcomes of y):

$$p(x) = \sum_{i=1}^{n} p(x|y_i)\, p(y_i),$$

where p(.) represents a probability density function. In statistics, this formula is known as the expected conditional probability density of x given y_i, i=1,2...,n, with $p(y_i)$ as the probability density of the conditional probability $p(x|y_i)$. It is a general measure of the degree of reasonable belief about the uncertain behaviour of variable x, given all possible outcomes of the other variables (y for example).

Example

The following attempts to illustrate the calculation of the marginal probability and the interrelations between the marginal, conditional, and joint probabilities. (A technical report precisely defining these probabilities is available on request from the Economics and Forecasts Division.)

Suppose there are three variables in an economic model, namely GDP (gross domestic product), CPI (consumer price index), and UR (unemployment rate). Suppose also that there can only be two states (outcomes) for CPI growth and UR respectively in some period: 4% to 6% and 9% to 11% for CPI; and 5% to 7% and 10% to 12% for UR. Thus, four joint states describe all the possible outcomes for CPI and UR (see column 1 and column 2).

States	CPI Growth (1)	UR (2)	Joint Probability of CPI & UR (3)	Conditional Prob of GDP being between 2% and 4%, given CPI & UR (4)	Joint Prob of GDP, CPI, & UR (5)
State 1	4%-6%	5%-7%	30%	15%	4.5%
State 2	4%-6%	10%-12%	40%	10%	4.0%
State 3	9%-11%	5%-7%	10%	25%	2.5%
State 4	9%-11%	10%-12%	20%	20%	4.0%

Column 3 shows the joint probability of CPI and UR for each state. For example, the probability of CPI growth between 4% and 6% and UR between 5% and 7% is 30%. Column 4 shows the conditional probability of GDP being between 2% and 4%, given a specified scenario of CPI and UR. For instance, under the circumstances in which CPI growth is between 4% and 6% and UR between 5% and 7%, the probability of GDP growing in the range of 2% to 4% is 15%.

The probability of GDP growing between 2% and 4%, given all possible states of CPI growth and UR, is called **the marginal probability**. The marginal probability gives due weights to all the possible precedent conditions leading to an outcome of GDP growth (for example, between 2% and 4%), and represents the total probability of that outcome. It is calculated by applying the joint probabilities (of CPI and UR) of all the possible precedent states to the conditional probabilities of the outcome. In this case, it is calculated as the weighted average of the four conditional probabilities (column 4), with the joint probabilities in column 3 serving as weights:

$$15\% \times 30\% + 10\% \times 40\% + 25\% \times 10\% + 20\% \times 20\% = 15\%$$

It means that the forecaster believes that there is, on average, a 15% probability that GDP growth would lie between 2% and 4%. This marginal probability associated with GDP should therefore be used separately (or independently) from the marginal probabilities of other variables in the model.

Furthermore, based on the information in the table, the joint probability of GDP, CPI and UR can be calculated. Hence, the joint probability of GDP being between 2% and 4%, CPI between 4% and 6%, and UR between 5% and 7% is 4.5% (that is, 30% x 15%). The individual (marginal) probability of GDP being between 2% and 4% can also be calculated by adding up all the joint probabilities in column 5.

In conclusion, the probabilities presented in this document are marginal probabilities. The marginal probability of a particular variable can be independently used to describe the uncertain behaviour of that variable. The joint use of these probabilities is possible if the joint probability or the correlations between the variables are available. However, information concerning the correlations between variables contained in this document is not fully available, and the user might wish to consult the Economics and Forecasts Division about it.